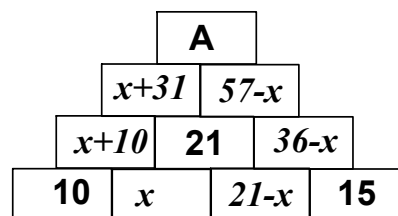


# SAMO 2010 – Junior First Round SOLUTIONS

1. B.  $4 \times 5 + 3 \times 6 = 20 + 18 = 38$
2. D. Since 5 is a factor the number must end in either 5 or 0. Since there is no factor of 2, the last digit cannot be 0 so must be 5
3. C.  $19 \div 11 = 1,72727\dots$  which uses 3 different digits
4. C. The prime possibilities are 23; 37; 53; 73
5. B.  $45^2 = 2025$  and  $2025 - 2010$  is 15
6. A.  $5^{104} \times 4^{52} = 5^{104} \times 2^{104} = 10^{104} = 1000\dots$  (i.e. 1 followed by 104 zeros) so sum of the digits is 1
7. D. With the OR as one unit (in two ways) we have four things to arrange. So number of possibilities is  $2 \times 4 \times 3 \times 2 = 48$
8. A. 
$$\frac{11! - 9!}{11! + 9!} = \frac{11 \cdot 10 \cdot 9! - 9!}{11 \cdot 10 \cdot 9! + 9!} = \frac{(11 \cdot 10 - 1) \cdot 9!}{(11 \cdot 10 + 1) \cdot 9!} = \frac{110 - 1}{110 + 1}$$
9. E. If the jug has capacity  $x$  ml then  $80\% \times 60\% \times x = 192$ .  
So  $0,48x = 192$  and  $x = 192 \div 0,48 = 4 \times 48 \div 0,48 = 400$

10. A. PM = 1 unit. The shaded trapezium's area is  
(average of parallel sides)  $\times$  (distance between parallel sides)  
 $= (1 + 6) \times \frac{1}{2} \times 4 = 14$

11. A. Putting  $x$  into the left empty cell on the bottom, so that the other empty cell must contain  $21-x$ , we work upwards and then have  $A = x + 31 + 57 - x = 88$ .



12. A.  $a = 6b$  and  $4b = c$  and  $a + c = 30$ . Therefore  $6b + 4b = 30$  and  $b = 3$
13. E. Suppose the square starts with sides of length  $x$  cm. Because of the smaller squares being removed, the base of the box has sides of length  $x - 2 \times 5$ , and therefore the volume of the box is  $5(x - 10)^2$ . But then  $(x - 10)^2 = 121$ , so  $x - 10 = 11$  and thus  $x = 21$ .
14. E. If the distance between Apetown and Beeville is  $x$  km, then the first speed is  $\frac{x}{2}$   
and the second speed is  $\frac{x}{3,2}$ . The difference between these is 30, so  
$$\frac{x}{2} - \frac{x}{3,2} = 30$$
, and thus  $(3,2 - 2)x = 2 \times 3,2 \times 30 = 192$ , so  $x = 192 \div 1,2 = 160$ .

**OR**

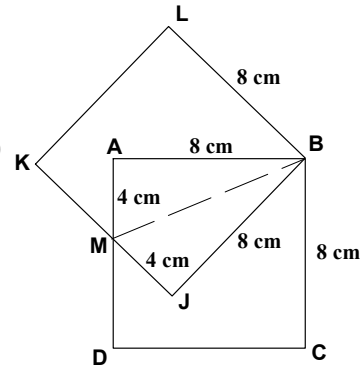
if  $v$  is the first speed, then we seek the value of  $2v$ , and we know that  $3,2 \times (v - 30) = 2v$ . This gives  $v = 3,2 \times 30 \div 1,2 = 80$ , so the required distance is 160 km.

15. B.  $BD + DC$  must exceed  $BC$ . Also  $DB + BC$  must exceed  $DC$ . Thus  $10 > 2x + 1 > 4$ . Then  $\frac{9}{2} > x > \frac{3}{2}$ , and since  $x$  is an integer, it must be 4 or 3 or 2, so sum of possible values is 9.

16. C. Figure 1 = 4 matches =  $1 \times 4$   
 Figure 2 = 10 matches =  $2 \times 5$   
 Figure 3 = 18 matches =  $3 \times 6$   
 Figure 4 = 28 matches =  $4 \times 7$   
 .....  
 Figure 20 =  $20 \times (20 + 3) = 460$

17. C. If the number of boys is  $x$ , the number of girls is  $100 - x$ . Then considering A symbols gives  $0,40x = 0,50(100 - x) + 4$ , so that  $0,4x = 50 - 0,5x + 4$ , giving  $0,9x = 54$  and so  $x = 60$ .

18. C. Area of hexagon =  
 $2 \times (\text{area of square}) - 2 \times \text{area of } \triangle ABM$   
 $= 2 \times 8 \times 8 - 2 \times \frac{1}{2} \times 4 \times 8$   
 $= 128 - 32 = 96 \text{ cm}^2$



19. D. Let the even numbers be  $2n, 2(n-1), 2(n-2), \dots, 2(n-10)$ . Their sum is  $(2n + 2n + 2n + \dots) - 2(1 + 2 + 3 + \dots + 10)$   
 $= 11 \cdot 2n - 2 \cdot \frac{1}{2} \cdot 10 \cdot 11 = 11 \cdot 2n - 11 \cdot 10 = 11(2n - 10)$ .  
 Since this is  $p$ ,  $2n - 10 = \frac{p}{11}$ , and then  $2n = \frac{p}{11} + 10$ .

$1 + 2 + 3 + \dots + k =$ $\frac{1}{2} k(k + 1)$ <p>(see formula sheet)</p>
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OR

Let the even numbers be  $m - 10, m - 8, m - 6, \dots, m + 6, m + 8, m + 10$ . Then their sum is  $11m$  and is also  $p$ , so the largest is  $\frac{p}{11} + 10$

20. C. Let A mean “Anne is telling the truth”,  $\bar{B}$  mean “Barbara is lying” etc. Then either  $A \Rightarrow \bar{B} \Rightarrow C \Rightarrow \bar{D}$  and two girls (Barbara and Diane) are lying or  $\bar{A} \Rightarrow B \Rightarrow \bar{C} \Rightarrow D$  and two girls (Anne and Catherine) are lying.